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Optimum Refractive-Index Difference for Graded-Index Fibers Resulting From Concentration-Fluctuation Scattering

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Since the use of graded-index optical-fiber waveguides can greatly reduce pulse dispersion, fibers with greater acceptance angles can be used to increase the power coupled from an incoherent source. The increase in acceptance angle is obtained by increasing the total index variation between the core and cladding, which is achieved by increasing the doping of the fiber. This paper shows that the increased loss of the fiber resulting from concentration-fluctuation scattering by the dopant eventually decreases the light at the output end of the fiber. Therefore, there is an optimum magnitude for the total index variation based on loss considerations alone. Analytic expressions are developed using a simplified theory of concentration-fluctuation scattering that assumes the dopant molecules are randomly distributed. These expressions are applied to doped-silica-based fibers, assuming typical values for the source intensity, fiber loss, and receiver sensitivity.

I. INTRODUCTION

As is well known, multimode optical-fiber waveguides with graded refractive-index profiles can have appreciably lower mode dispersion than fibers with a step index profile for the same modal content.[1–3] One consequence of this fact is that, for a given bit rate and fiber diameter, a graded-index fiber can be used that propagates many more modes. This is important when the source is incoherent, such as a light-emitting diode, since the amount of power that can be coupled into the fiber from such a source is proportional to the number of modes it can propagate.

At what distance a signal of a given bit rate must be regenerated depends on how much dispersion has taken place and on the strength of the signal. At first, it seems that the distance would be maximized by increasing the number of modes until the maximum distance resulting from dispersion is equal to that resulting from signal strength. In this paper, we show that this may not be the case, and that a maximum distance can arise from signal strength considerations alone.

Briefly, the argument is as follows. The refractive index of the fiber is graded by a graded concentration of a dopant in a base glass. To increase the number of modes propagating in a fiber of a given diameter, the magnitude of the index variation must be increased and, therefore, so must the dopant concentration. Associated with the dopant is an additional loss resulting from scattering by fluctuations in the composition of the glass.[4] Therefore, we have a situation in which, as the number of modes is increased, the loss coefficient is also unavoidably increased. The competition between the two effects produces an optimum number of modes and, hence, a maximum distance between repeaters.

In this paper we derive the details of this effect and we calculate its magnitude for typical source, fiber, and receiver characteristics, using a simplified theory of concentration-fluctuation scattering[4] that assumes a random distribution of dopant molecules and neglecting any effect of changes in the physical properties and fictive temperature of the glass on density fluctuations.

II. THEORY

We consider a fiber with a cylindrically symmetric refractive-index profile

$$n = \begin{cases} n(r) & r < a \\ n_0 & r \geqq a, \end{cases}$$

where n_0 is a constant. For light guidance, $n(r) > n_0$.

For definiteness, we assume that the dopant increases the refractive index of the base glass, since this is generally the case for fibers based on fused silica.* It has been shown[4] that, assuming the dopant molecules are randomly distributed, a glass doped to produce an index n from a host glass of index n_0 will have an additional loss resulting from scattering by concentration fluctuations given by

$$\alpha_c = A \frac{n - n_0}{n_0}, \tag{1}$$

* Although it has been shown (Ref. 5) that boron oxide can be used to lower the refractive index of fused silica, this is the only dopant that does so.

where α_c is the excess loss coefficient and A is dependent on the refractive properties of the dopant and the host glass and on the inverse fourth power of the wavelength. Strictly speaking, the assumption of a random distribution of dopant molecules limits the validity of eq. (1) to dilute dopant concentrations. There is, however, experimental evidence that, in the Na_2O—CaO—SiO_2 and TiO_2—SiO_2 glass systems, it is valid up to the concentrations of interest[4] and, although its validity for other glass systems of interest has not been established, it does represent a reasonable starting point for estimating the effect of concentration fluctuation scattering. If we neglect any changes in the density fluctuation scattering because of changes in the physical properties and fictive temperature with doping, α_c is the total increase in scattering loss.

If $I(r)$ is the power flow per unit area in the fiber, then from eq. (1) the excess loss coefficient of the fiber will be[6]

$$\bar{\alpha}_c = \frac{\frac{A}{n_0}\int_0^\infty [n(r) - n_0]I(r)2\pi r dr}{\int_0^\infty I(r)2\pi r dr}. \tag{2}$$

Although the power flow in a particular mode of a cylindrically symmetric fiber generally has an azimuthal dependence, there is always another degenerate mode that can be expected to carry the same power and produce a total power independent of azimuth. $I(r)$ will be a function of the relative excitation of the different modes as well as of the index profile $n(r)$. For example, Gloge and Marcatili[3] have calculated $I(r)$ for the one-parameter class of index profiles,

$$n(r) = \begin{cases} n_0\left[\dfrac{1 - 2\Delta(r/a)^\gamma}{1 - 2\Delta}\right]^{\frac{1}{2}} & r < a \\ n_0 & r \geqq a, \end{cases} \tag{3}$$

where the parameter $\gamma \geqq 1$. This class includes the parabolic profile $\gamma = 2$ and the step profile $\gamma = \infty$. Assuming that: (*i*) the fiber can propagate many modes, (*ii*) $\Delta \ll 1$; (*iii*) $n(r)$ is relatively constant over distances of a wavelength; and (*iv*) all modes carry the same power, Gloge and Marcatili[3] have shown that $I(r)$ for the above class of index profiles is

$$I(r) = \begin{cases} I_0[1 - (r/a)^\gamma] & r < a \\ 0 & r \geqq a. \end{cases} \tag{4}$$

Multimode fibers of interest will undoubtedly satisfy assumptions (*i*) to (*iii*). Assumption (*iv*) is less realistic but, as the actual distribution of power among the modes of a given fiber will depend on the relative loss of the modes and the coupling between them, it provides a convenient first approximation.

Since we are restricted to $\Delta \ll 1$ by assumption (*ii*), it is reasonable to make the approximation to the profiles of eq. (3)

$$n(r) \approx n_0 \left[\frac{1 - \Delta(r/a)^\gamma}{1 - \Delta} \right] \quad r < a. \tag{5}$$

Using eqs. (4) and (5) in eq. (2), we obtain

$$\bar{\alpha}_c = \frac{A\gamma}{\gamma + 1} \frac{\Delta}{1 - \Delta} \approx \frac{A\gamma}{\gamma + 1} \Delta. \tag{6}$$

We are interested in considering only those index profiles that have low pulse dispersion. From among the class of profiles of eq. (3), Gloge and Marcatili[3] found that the lowest dispersion occurs for a value of γ very close to 2. Therefore, we will consider specifically the parabolic profile for which

$$\bar{\alpha}_c = \tfrac{2}{3} A \Delta \tag{7}$$

from eq. (6).

III. APPLICATION TO REPEATER SPACING

We now consider the consequences of the Δ-dependent loss of eq. (7) on the distance one can transmit before the signal becomes so weak that it must be regenerated. The loss of the fiber is

$$10 \log_{10} \frac{P_i}{P_0} = (\alpha_0 + \bar{\alpha}_c)L, \tag{8}$$

where P_i is the power at the input, P_0 is the power at a distance L kilometers from the input, $\bar{\alpha}_c$ is the fiber loss coefficient in dB per kilometer because of concentration-fluctuation scattering, and α_0 is the fiber loss coefficient in dB per kilometer resulting from all other scattering and absorption processes.

For incoherent excitation of the fiber, P_i will be a function of Δ. An incoherent source is characterized by a brightness B that is the power per unit solid angle per unit area. In general, B is a function of both position and direction. Since the area of the fiber and the range of directions over which the fiber accepts light are small, we assume B constant. The solid acceptance angle at the fiber end face is[3]

$$\Omega(r) = \pi[n^2(r) - n_0^2] = \frac{2\pi n_0^2 \Delta}{1 - 2\Delta} \left[1 - \left(\frac{r}{a} \right)^\gamma \right]$$

and, therefore, the power coupled to the fiber neglecting any reflection loss will be

$$P_i = \int_0^a B\Omega(r)2\pi r dr = 2\pi^2 a^2 n_0^2 B \left(\frac{\gamma}{\gamma + 2}\right) \frac{\Delta}{1 - 2\Delta} \approx C\Delta. \quad (9)$$

The maximum distance of transmission will be that value of L for which P_0 is equal to the minimum detectable signal as determined by the receiver sensitivity. Combining eqs. (7), (8), and (9), we obtain for the maximum transmission distance

$$L = \frac{10 \log_{10} (C/P_{0\,\min}) + 10 \log_{10} \Delta}{\alpha_0 + \frac{2}{3} A \Delta}. \quad (10)$$

This function has a maximum value as a function of Δ for any given choice of C, $P_{0\,\min}$, α_0, and A, and, therefore, from loss considerations alone there is an optimum Δ.

In Fig. 1, L is plotted as a function of Δ for a possible dopant of fused silica, TiO_2,[7] and for three values of $C/P_{0\,\min}$ (a ratio that is the total allowable loss normalized by the fractional index variation Δ) that cover the range expected for a practical system. They correspond to $P_{0\,\min}$ between 10^{-5} and 10^{-6} milliwatts[8] and C between 1 and 10 milliwatts (P_i between 10 and 100 microwatts for a fiber with $\Delta = 0.01$).* The values used for the other parameters in eq. (10) are $\alpha_0 = 4$ dB per kilometer, corresponding to the loss of the best Corning fibers in the vicinity of 0.85 micrometer,[10] and from the calculations of Ref. 4 a value of A of 170 dB per kilometer, corresponding to TiO_2 at a wavelength of 0.90 micrometer. The maxima are quite broad; this (together with the result from Fig. 2 that the optimum Δ is not a strong function of receiver sensitivity) means that a particular choice of Δ near the optimum will give repeater spacings near the maximum over a considerable range of receiver sensitivities.

Another interesting point from Figs. 1 and 2 is the relatively low value of the optimum Δ, which lies in the range of 0.0037 to 0.0064 for the expected source, receiver, and fiber parameters.

IV. CONCLUSIONS

The purpose of this paper has been to show that, when concentration-fluctuation scattering is taken into account, pulse dispersion does not necessarily determine the total index variation that will maximize

* Burrus and Dawson (Ref. 9) have shown that it is possible to make diodes with $C = 10$ milliwatts and long life. To cover the full range of possibilities, however, we also include the conservative value $C = 1$ milliwatt.

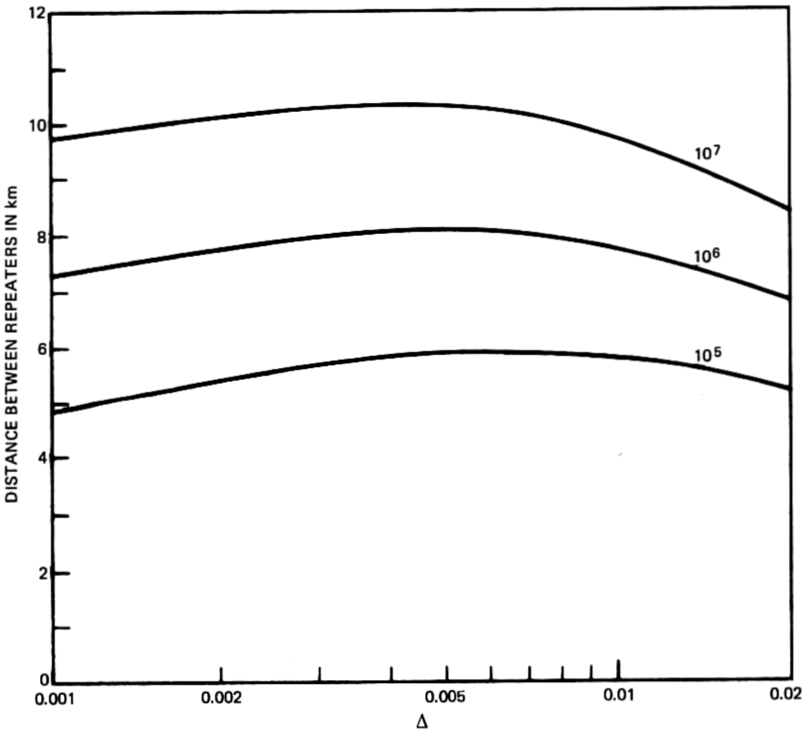

Fig. 1—Plot of the maximum distance between repeaters resulting from signal strength as a function of the total relative index variation Δ of a TiO_2-doped fused-SiO_2 parabolic profile graded-index optical waveguide. The numbers labeling the curves are the values of $C/P_{0\,min}$, the ratio of the power available from the source normalized by Δ [which, from eq. (9), is a function of the brightness of the source and the diameter and index profile of the fiber] to the minimum power the receiver can detect.

the distance a signal can be transmitted over a graded-index optical-fiber waveguide. We have specifically considered the parabolic index profile, since it is close to the optimum for minimizing pulse dispersion. A theory of concentration-fluctuation scattering has been used that assumes a random distribution of dopant molecules, and changes in density fluctuation scattering have been neglected. With these assumptions, we find for TiO_2 doping and typical source, receiver, and fiber parameters that the value of Δ maximizing the distance between repeaters from loss considerations is in the range of 0.0037 to 0.0064. The corresponding maximum distance is in the range of 10.3 to 5.9 kilometers.

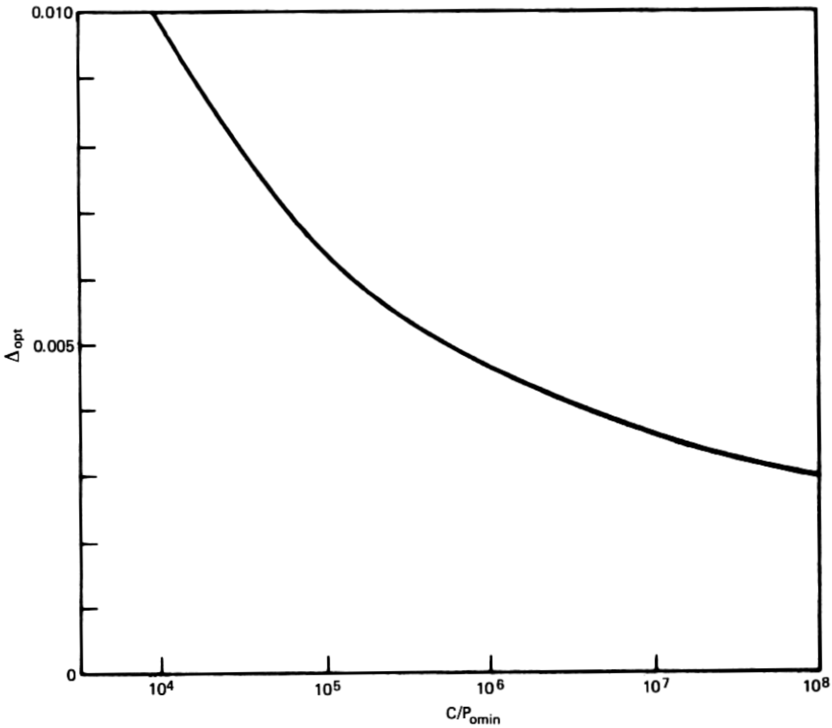

Fig. 2—Plot of Δ_{opt}, the value of the total relative index variation of a parabolic profile graded-index optical waveguide that maximizes the distance between repeaters because of signal strength, as a function of $C/P_{0\,min}$ for TiO_2-doped fused SiO_2.

Since the pulse dispersion τ for a parabolic index profile is approximately $(Ln_0/c)(\Delta^2/2)$,[3] we find, for $\Delta = 0.0047$ and $L = 8.1$ kilometers (the optimum values for $C/P_{0\,min} = 10^6$), that $\tau = 0.44 \times 10^{-9}$ seconds corresponding to a bit rate of 2.3 gigabits per second. Hence, for a system having $C/P_{0\,min} = 10^6$ and a lower bit rate, the upper limit on Δ and the maximum repeater spacing would be determined by loss considerations alone.

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